Optimization

Math 130 - Essentials of Calculus

18 November 2019

Starting Example

Example

A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. She needs no fence along the river. What are the dimensions of the field that has the largest area?

Procedures for Solving Optimization Problems

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4. *Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.*
5. *Determine the desired maximum or minimum value using calculus.*

Now You Try It!

Example

Find the dimensions of a rectangle with perimeter 100*m whose area is as large as possible.*

Example without a Feasible Domain

Example

Find the dimensions of a rectangle with area $1000m^2$ *whose perimeter is as small as possible.*

Now You Try It!

Example

A box with a square base and open top must have a volume of $32,000 cm^3$*. Find the dimensions of the box that minimize the amount of material used.*

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2. *Find two positive numbers whose product is* 100 *and whose sum is a minimum.*
3. *Find a positive number such that the sum of the number and its reciprocal is as small as possible.*